

Foundations of Query Languages
Summerterm 11
Discussion by 29.06.2011

3. First Order Queries

Exercise 1 (FO Undecidability)

Formulate conjunct of subformula ϕ_{rest} in the proof of Trakhtenbrot's theorem.

Exercise 2 (FO Undecidability)

- a) At the end of conjunct e of ϕ_{rest} , we state that $\forall r \neq p(T_0(r, t') = T_0(r, t))$. What does this mean?
- b) Why isn't it necessary to state in addition $\forall r \neq p(T_1(r, t') = T_1(r, t))$?

Exercise 3 (FO^{pos})

Let FO^{pos} denote FO without negation but including the connectives conjunction, disjunction, existential and universal quantification, forbidding, moreover, the use of $\neq$, but allowing $=$ to occur in formulas. Consider FO^{pos} under the active domain interpretation.

- a) Does Trakhtenbrot's Theorem hold for FO^{pos} ?
- b) What if we added logical implication as a connective to FO^{pos} and allowed at the same time the use of $\neq$?
- c) What is the program complexity of FO^{pos} ? (Consider Boolean queries only!)

Exercise 4 (RA^+)

Recall RA^+ , called Positive Relational Algebra, consists of RA restricted to the operations selection, projection, cross product, rename, and union.

- a) Explain, why is the difference operator is a primitive operator in RA. Explain, why is it not possible to express the difference operator as a combination of operations selection, projection, cross product, rename, and union.
- b) Would the addition of the intersection operation alter the expressive power of RA^+ ?
- c) What is the relationship, in terms of expressive power, between RA^+ and FO^{pos} under the active domain interpretation?
- d) What is the program complexity of RA^+ ?

Exercise 5 (FO)

Let R be a relation and let X be a set of attributes of R and A a single attribute of R . A functional dependency $X \rightarrow A$ expresses that for any two tuples t_1 and t_2 in R , whenever t_1 and t_2 have equal X -values, then they also have equal A -values. A set of attributes X of R is called a superkey of R if $X \rightarrow A$ for each attribute A of R . Assume the schema of R is (A, B, C, D, E) . Find an FO query that states that the functional dependency $AB \rightarrow E$ holds in R and another one stating that DE is a key of R .